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Superheavy Elements at High Angular Momenta

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Abstract: Potential-energy surfaces at high spin are calculated for nuclei in the superheavy region by use of the cranked Nilsson-Hamiltonian. It is found, that the maximum fission barrier is shifted from the doubly magic $^{298}_{114}$ to nuclei with fewer protons and more neutrons, as the spin increases. We also estimate, by use of simple models, the half-lives associated with alpha-decay, gamma-decay and fission as functions of spin along the yrast line. As a representative example a detailed discussion of the yrast spectrum of $^{298}_{114}$ is presented.



Many studies ¹⁻³⁾ during the past 10 to 15 years have suggested the possibility of an "island" of relatively long-lived superheavy elements around $Z=110$ and $N=184$. The predictions for the half-life of the most stable element, $^{294}_{110}$, in this region usually fall in the range $1-10^{10}$ years. Although these estimates could be wrong by a factor of 10^{10} or more, the relatively long half-lives that were predicted for elements in this region stimulated many searches for superheavy elements in nature ⁴⁾.

However, no superheavy elements were found. Consequently, the search for them in nature has tapered off and the effort to find them is now concentrated on their artificial production. Here two possible methods were originally thought to be particularly promising.

One of the methods considered was multiple neutron capture by heavy elements, followed by beta decay. However, in the neutron flux generated by exploding atomic bombs no element with $A \geq 260$ appears to have been produced ⁵⁾. This lack of elements with $A \geq 260$ is accounted for by many different calculations, which all show that for these elements the fission barrier is lower or of the same magnitude as the neutron separation energy ⁶⁻⁸⁾.

The other of the methods considered for artificially producing superheavy elements was production by means of some suitable heavy-ion reaction ⁹⁾. In a heavy-ion collision a superheavy compound system may be produced by complete fusion of target and projectile, by transfer reactions or in deeply inelastic events. For complete fusion events one expects from theoretical considerations that the cross-section for the production of a superheavy compound system is severely limited by entrance channel effects ⁹⁻¹¹⁾.

Experimentally, so far, no superheavy element has been identified in a complete fusion event or in any other type of experiment.

Many unique decay characteristics may be used to identify superheavy elements. For example, it has been estimated that in fission of a superheavy atom about 10 neutrons are released ¹²⁾. Also α -decay chains, leading to a relatively stable superheavy element from a more unstable superheavy element formed in a heavy ion reaction far from the center of the superheavy island, should be easily identifiable. Some examples of such decay chains are given in refs. ^{2,13)}.

However, the above theoretical studies all refer to the decay of nuclei in their ground state. A superheavy compound system formed in a heavy-ion reaction would be considerably excited above the ground state and carry a fairly high angular momentum. An investigation of the stability of superheavy elements toward fission at high angular momenta is performed in refs. ^{14,15)} and at high temperatures in ref. ¹⁵⁾. The deexcitation to the yrast line takes

place under competition between fission, particle evaporation and gamma decay. If the nucleus reaches the yrast line there is still a competition between these different decay modes. The fission barrier of $^{298}_{114}$ is, even at as high an angular momentum as $I=60$, calculated ¹⁴⁾ to be about 4 MeV, providing the whole (ϵ, γ) deformation plane is accounted for. In this letter a more detailed investigation of the yrast properties in the superheavy region is reported.

In this investigation potential energy surfaces in the (ϵ, γ) plane, with an implicit inclusion of ϵ_4 in the rotating liquid drop part, are calculated at varied angular momenta. The calculation is based on a Strutinsky-renormalized, cranked Nilsson Hamiltonian as described in refs. ^{14, 16)}. The single-particle potential parameters used are the so called SPL-2-parameters ³⁾, which have been determined by extrapolation from known spherical nuclei. From the potential-energy surfaces many properties can be extracted. One may for example determine ground state deformations and fission barrier heights and learn whether the yrast band is of collective or single-particle type.

Hypothetical nuclei around the doubly magic $^{298}_{114}$ are expected to be very similar to those around $^{208}_{\text{Pb}}$. Pairing, for example, is expected to play a minor role and is not included in the present calculation. In the lead region, the yrast states are build by single-particle excitations, that is rotation around the symmetry axis, and isomeric high-spin states are likely to occur ¹⁷⁾. This is in the present investigation also found to be the case for many nuclei in the superheavy region.

At spin zero the equilibrium deformation is calculated to be almost spherical for most nuclei studied in fig. 1 with $N \geq 182$. When the potential is cranked, the negative shell energy associated with proton number 114 and neutron number 184, decreases because of the high energy cost of exciting a nucleon across the gap in a magic nucleus. However, with some particles (or holes) added to the spherical $^{298}_{114}$ it is possible to maintain a part of the static shell energy. The particles are thus, at a low energy cost, placed in orbitals with high m -values in order to get a high total angular momentum. This polarizes the core and usually gives rise to an oblate or, in the case of holes, a prolate equilibrium deformation. However, for the heavy nuclei studied here, the deformation changes due to polarization are expected to occur at higher spins than in lighter nuclei. When high j -shells in the vicinity of the gap are involved, a large amount of angular momentum may be obtained at a low energy cost. For moderate spin values a large negative shell energy is therefore expected to be connected with a spherical shape for nuclei with $N > 184$ (as the $(h\ 11/2)_n$ shell situated just above the $N=184$ -gap will then be occupied) and $Z < 114$ (as the $(i\ 13/2)_p$ and $(f\ 7/2)_p$ shells

below the $Z=114$ -gap will then be emptied). (See also figs. 1a and 2a in ref. ¹⁷⁾.) These conclusion are verified by the calculations, as presented in fig. 1. It is found that, also at as high spin as 40, nuclei with the above mentioned proton and neutron numbers, are rotating around the spherical symmetry axis.

The nuclei with $N \approx 176-180$ are at spin zero calculated to have a small oblate or triaxial deformation ($0.05 \leq |\epsilon| \leq 0.15$). At spin 20 and 40 most of them are found to rotate around the axis with the classically smallest moment of inertia. The rotation is collective and yrast isomers are highly unlikely to occur in these nuclei.

In fig. 1 the height of the fission barrier is plotted versus proton and neutron numbers for the spin values $I=0, 20$ and 40 . A comparison of the $I=0$ figure with the very similar fig. 5b in ref. ³⁾, where also pairing is accounted for, shows a very similar behaviour of the fission barrier heights with a discrepancy of at most 2 MeV. At spin zero the fission barrier is largest for the doubly magic nucleus $^{298}_{114}$. For spin values different from zero, it is found that it is no longer this nucleus that has the highest fission barrier in this region. Fig. 1 shows, for example, that the nucleus $^{302}_{114}$ has a lower fission barrier than $^{298}_{114}$ at $I=0$, while it has a 0.7 MeV higher barrier at $I=20$. The main reason for this shift is the change of the shell energy with increasing spin as discussed above. Nuclei with $N \geq 184$ and $Z \leq 114$ are thus at high angular momenta calculated to have the highest fission barriers in the mass region considered.

For higher spin values, the centrifugal force usually cuts down the fission barrier. However, in certain cases strong shell effects cause an increase of the barrier with increasing spin. For example, this is the case for $Z=106$ and $N=186$ (fig. 1). This is due to the very low energy cost required to achieve spin values up to the aligned $I=34$, in this almost spherical nucleus with the configuration $(i\ 13/2)^{-8}_p (h\ 11/2)^2_n$.

When the rotation is calculated to take place around a symmetry axis, it is of interest to look in more detail at the yrast states as the occurrence of high spin isomers now is possible. The scheme applied here for calculating yrast traps was developed in ref. ¹⁴⁾. A large number of particle-hole configurations are investigated for a given spin. Each configuration is minimized with respect to the deformation (both ϵ and ϵ_4 are here included in the shell energy part). The calculated yrast line for $^{298}_{114}$ is shown in fig. 2. A very rough estimate of the gamma half-lives is performed in the same way as in ref. ¹⁸⁾. Isomeric states are marked by arrows in fig. 2 and can be found at remarkably high spins.

In the case of rotation around a symmetry axis one may extract an averaged moment of inertia by fitting a parabola to the yrast line in a certain spin region. It is then to be noted that this averaged moment of inertia, if fitted in the spin region of $I=0-20$, is only about 30% of the rigid body value (see also ¹⁹⁾). This large deviation is a pure shell effect. It arises because in the doubly magic nucleus particles have to be excited across the gaps in order to build up angular momenta. The calculated moment of inertia for ²⁹⁸114 increases with spin, and a fit in the spin region $I=50-70$ gives 120% of the rigid body value determined for the calculated deformation in this spin region. The lower part of fig. 2 shows the equilibrium deformation at each spin. At spins 43-46 a shape transition occurs from sphericity to an oblate deformation with $\epsilon \approx -0.2$. The rigid moment of inertia in the oblate region is about 13% larger than the spherical rigid moment of inertia. This is to be compared with the much larger fluctuations in the averaged moment of inertia caused by shell effects. From this point of view it seems to be difficult to extract the deformation of a nucleus, by a measure of the "moment of inertia", when the rotation takes place around a symmetry axis ¹⁹⁾.

If a compound nucleus has reached the yrast line, one should note that also here other decay modes than electromagnetic transitions might occur. In the $I=0$ ground state, the nucleus ²⁹⁸114 is calculated ³⁾ to have a fission half-life of 10^8 years (providing the pairing is accounted for and ϵ_4 is included also in the shell energy part) and an alpha half-life of about 0.1 years. Above we saw that the fission barrier usually decreased with increasing spin. The probability of emitting an alpha particle from a high-spin yrast state will depend on the energy available for the alpha particle. This energy can be much larger in a high spin state than in the $I=0$ state, in particular when the alpha particle carries some spin. However, in addition to the Coulomb barrier, it then also has to penetrate a centrifugal barrier.

These two decay modes, alpha-decay and fission, seem to be those that can compete with the gamma transitions. To calculate the alpha half-lives at high spin a simple model of Gamow type, as presented for example in ref. ¹⁸⁾, is used. The fission half-lives are determined by use of a one-dimensional model ²⁾ that has been found successful in reproducing ^{20,21)} actinide spontaneous fission half-lives. For many nuclei studied here, the shapes become triaxial during the barrier penetration process. This occurs generally for nuclei with an oblate equilibrium deformation. The fission barrier for nuclei with a calculated spherical or prolate deformation, is here usually found to be lowered, by up to 5 MeV, as the deformation at the saddle point changes from

prolate to triaxial. For this reason the calculation of the fission coordinate has to be somewhat modified compared to refs. ^{20,21)}.

In a one-dimensional WKB approximation the penetrability P through the barrier is given by ²²⁾

$$P = (1 + \exp K)^{-1}$$

where

$$K = 2 \int_{r_1}^{r_2} \left\{ \frac{2B_r(r)}{\hbar^2} [V(r) - E_0]^{1/2} dr \right\}$$

Here the limits of integration r_1 and r_2 are the barrier entrance and exit points at a penetration energy E_0 that lies 0.5 MeV above the ground state. The one-dimensional fission barrier, $V(r)$, is determined from the calculated multidimensional surfaces. The function $B_r(r)$ is the mass parameter associated with motion in the fission direction. This parameter is related to the choice of fission coordinate r . As in refs. ^{2,20,21)} we choose as our coordinate r the distance between the centers of mass of the two halves of the nucleus. The division into halves is for each deformation done perpendicular to the longer of the principal axes. For the $(\epsilon, \epsilon_4, \gamma)$ parametrisation that is used in the Nilsson model, the expressions for r is determined by integrating numerically the appropriate expressions given in ref. ²³⁾. For the mass parameter $B_r(r)$ we use the model suggested by ref. ²⁾:

$$B_r(r) = k(B_r^{ir}(r) - \mu) + \mu$$

where μ is the reduced mass of the final fragments and k is a semiempirical constant. In the study of ref. ²¹⁾ k was determined to 11.5. The irrotational inertia is approximated by an expression given in ref. ²⁴⁾. Although the parameters in the expression for B_r were determined for spontaneous fission, we here use the same values also for $I \neq 0$. Some support for choosing the same fission mass parameter for $I \neq 0$ as for $I=0$ is given by ref. ²⁵⁾ which shows that in a pure oscillator single particle potential the mass parameter is essentially (in this model calculation the mass parameter was approximatively equal to the irrotational value) independent of the distribution of the nucleons over the single particle levels.

The half-lives corresponding to the three different decay modes of ²⁹⁸₁₁₄ are shown in fig. 3 as functions of angular momentum. Note the possibility of isomeric states also at very high spins and that some of these isomers have a calculated gamma half-life longer than the corresponding alpha

half-life. This phenomenon is discussed in ref. ¹⁷⁾ for much lighter nuclei, where some of the yrast traps should manifest themselves as alpha-emitting isomers. In the superheavy region, on the other hand, fission becomes the dominating decay mode already at moderately high spins. The calculated fission half-life is a rapidly decreasing function of angular momentum. The fission half-life approximately decreases by one power of ten for each unit of angular momentum in the region $I=20-40$. Spin $I \approx 40$ seems to be the highest angular momentum at which the cold nucleus would survive fission and decay to the $I=0$ ground state. A remarkable result is that although alpha decay is the dominating process in the $I=0$ ground state, no other state along the yrast line is calculated to emit an alpha particle.

For the slightly heavier elements as $^{302}_{114}$, the fission half-life decreases much more slowly as a function of angular momentum (fig. 1). In this nucleus the $I=0$ alpha half-life is only $10 \text{ s}^{3)}$. This means that alpha decay becomes a much faster decay channel than fission for spin values around 40. It would thus be possible that an yrast isomer in this spin-region emits an alpha particle instead of a gamma quanta. Such a delayed alpha particle would then carry an angular momentum of about $10 \hbar$ and an energy of about 14 MeV.

The above results are of interest to the attempts of forming superheavy elements in heavy-ion reactions. One should try to choose projectile-target combinations that lead to compound systems with as high fission barriers as possible at high spin. However, the present investigation shows that with increasing spin, the "center of the superheavy island" is displaced toward increasingly neutronrich nuclei and then become still harder to reach in heavy-ion reactions. On the other hand, even if the compound system does not deexcite to a relatively long-lived ground-state configuration, it may be possible to study the decay of yrast isomers by observing the energetic alpha particles that should sometimes be emitted.

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Figure captions

Fig. 1. Calculated fission barrier heights (in MeV) for the studied even-even superheavy nuclei at different angular momenta as obtained from potential-energy surfaces in the (ϵ, γ) plane. Note the change of maximal fission barrier from the doubly magic $^{298}_{114}$ to nuclei with lower proton numbers and higher neutron number. Regions where the rotation is calculated to take place around a symmetry axis is indicated by a "S" if spherical ($|\epsilon| \leq 0.05$), "p" if prolate and "O" if oblate. These regions are favourable for the formation of yrast traps. In the regions marked by a "C" the rotation is found to be of collective type.

Fig. 2. The calculated yrast line for $^{298}_{114}$. States (above spin 15) with relatively long estimated gamma-decay times are marked by arrows, whose lengths differentiate between orders of magnitude in the estimates as indicated in the figure. The abbreviation E.T. stands for "energy trap", which means that the state energetically is forbidden to gamma decay with a multipolarity lower than three. The lower part of the figure shows for each spin the equilibrium deformation ϵ (spherical or oblate) with an implicit minimization with respect to ϵ_4 .

Fig. 3. The estimated lifetimes of calculated yrast states in $^{298}_{114}$ for gamma-decay (circles), alpha-decay (jagged line) and for fission (steeply downsloping line). The arrows signify energy traps and the filled circles states with a relatively long estimated lifetime (>10 ns) with respect to gamma-decay. Note that the alpha-decay is calculated to be the fastest decay mode only at spin zero.

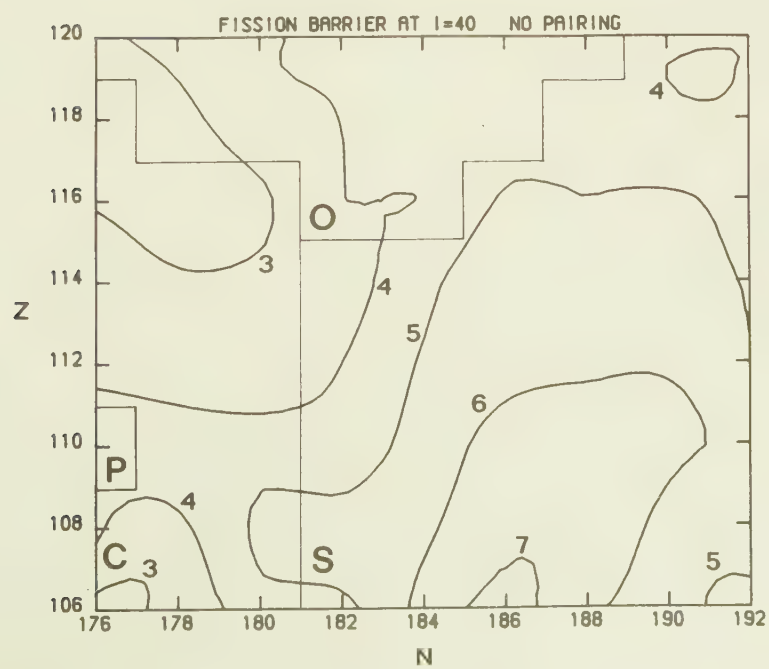
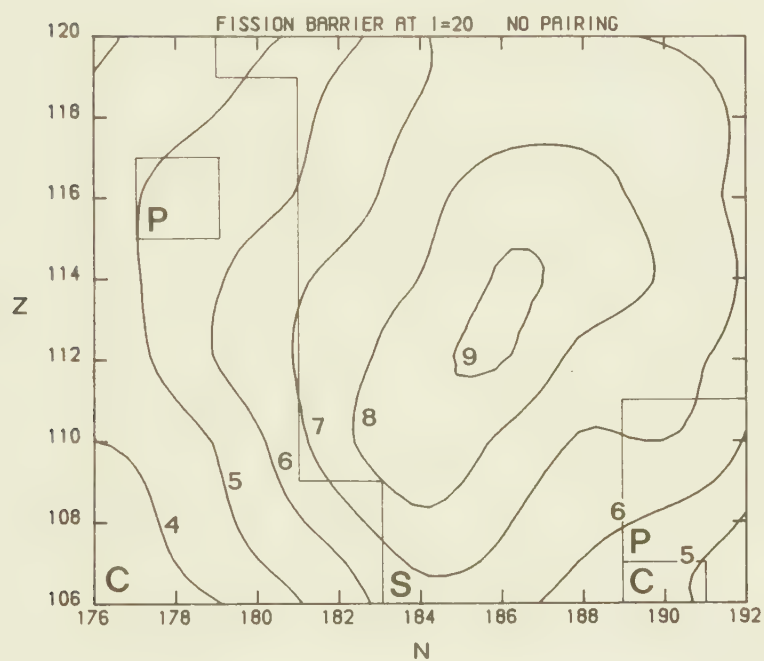
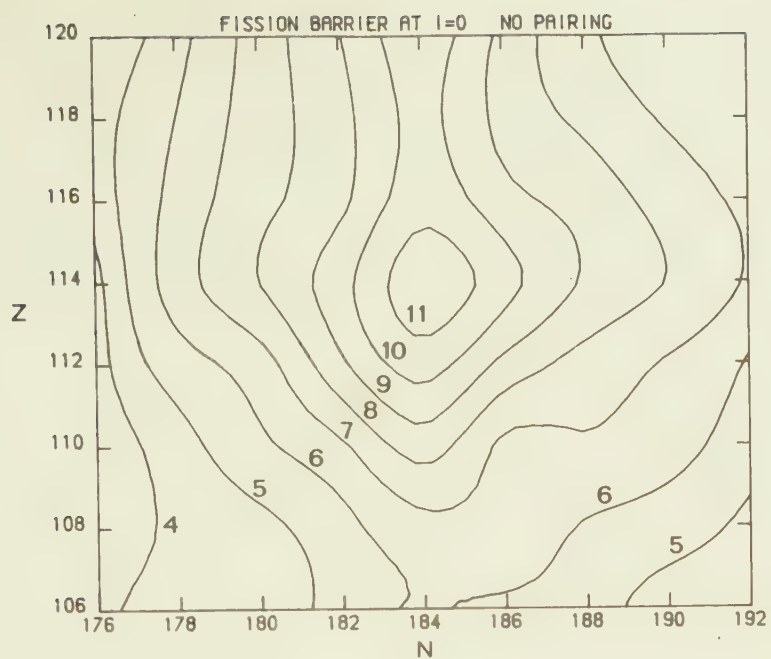


Fig. 1

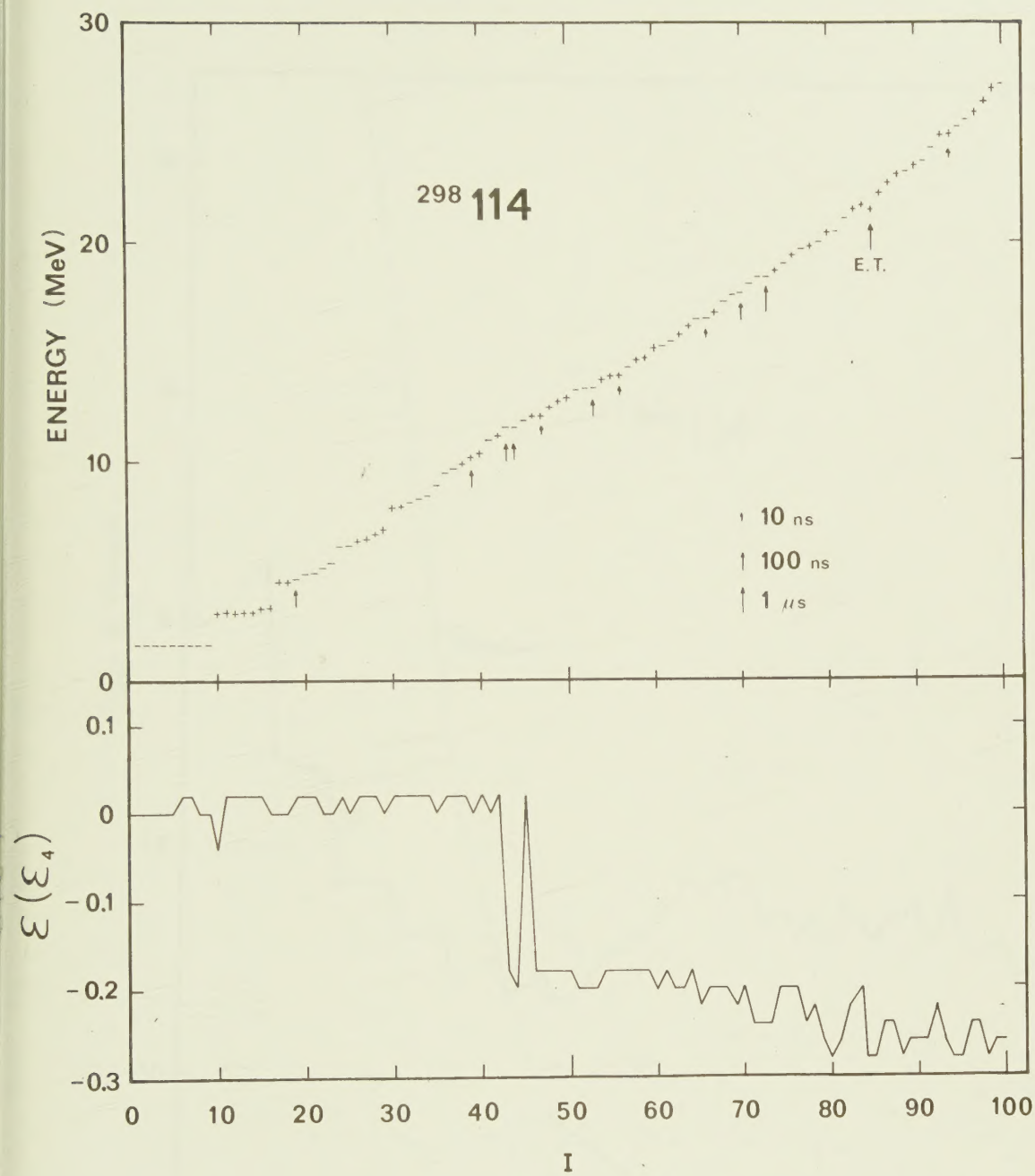


Fig. 2

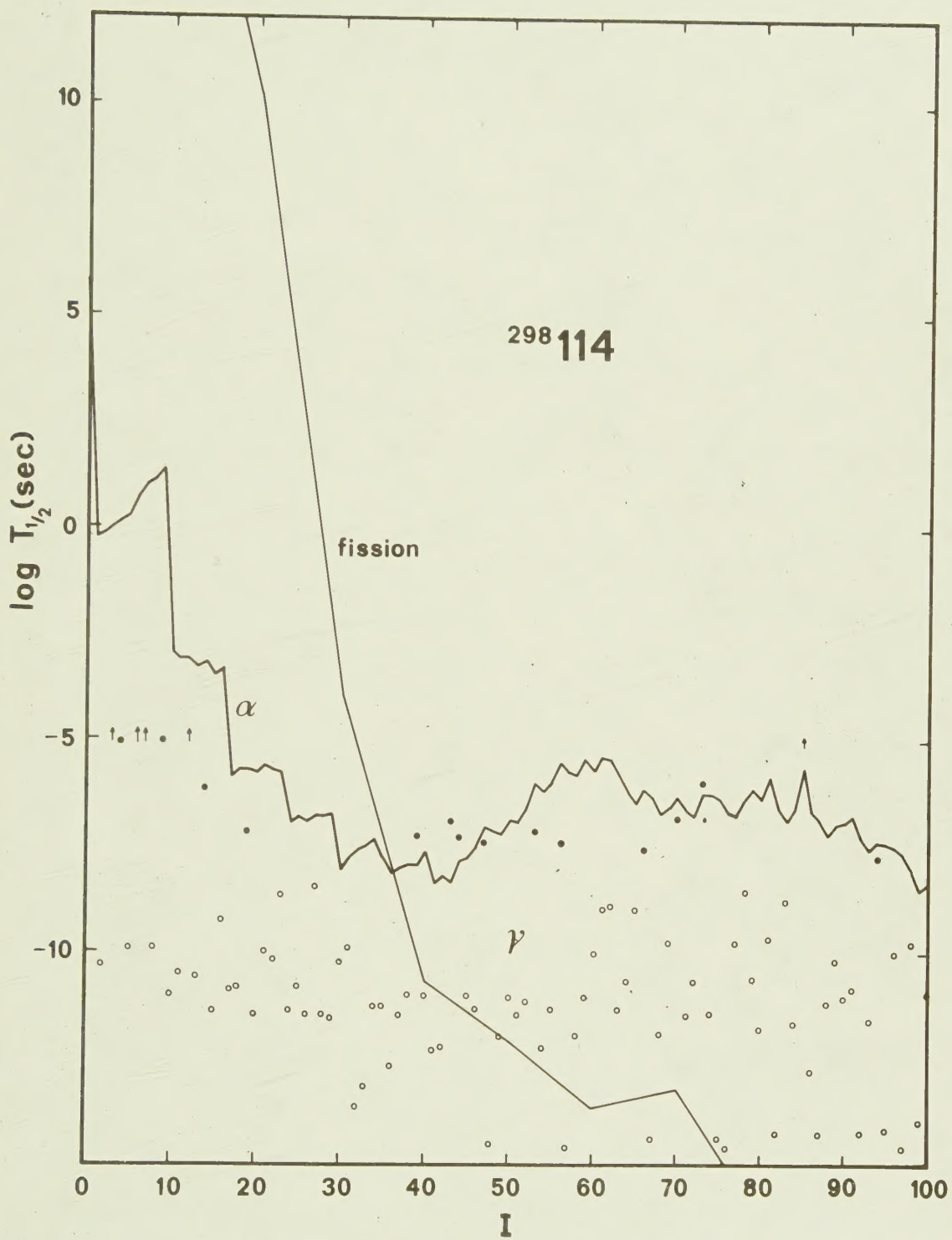


Fig. 3

